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Chaos for linear fractional transformations of shifts ☆

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We characterize chaos for $\varphi(B)$ on Banach sequence spaces, where φ is a Linear Fractional Transformation and B is the usual backward shift operator. Characterizations are computable since they involve only the four complex numbers defining φ.

Chaos has been usually considered a non-linear phenomena, although linear chaos may appear if we let the underlying space to be infinite dimensional. According to Devaney [1] and [23], a continuous map on a metric space is chaotic provided it is topologically transitive and it has a dense set of periodic points. This paper deals with the chaotic behaviour of a class of bounded linear maps (operators) defined on separable Banach spaces. Within this context, it is well known that topological transitivity is equivalent to the existence of a dense orbit, and this property is known for operators as hypercyclicity. Therefore, we have that operators defined on separable Banach spaces are chaotic if and only if they are hypercyclic and admit a dense set of periodic points. The monographs [2,3] are very good sources for the theory of linear dynamics.

The first example of a hypercyclic operator on a Banach space was given by Rolewicz in 1969 [4]. He proved that multiples λB of the backward shift operator $B(x_1, x_2, \dots) := (x_2, x_3, \dots)$ are hypercyclic on the space ℓ^1 of absolutely summable sequences if and only if $|\lambda| > 1$; in fact, they are chaotic. Since then, (weighted) shift operators defined on sequence spaces have become a usual ground to study linear dynamics. Hypercyclicity for weighted backward shifts defined on ℓ^p was characterized by Salas and he also showed

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that any perturbation of the identity by a weighted backward shift is always hypercyclic [5]. Following this direction, characterizations for hypercyclic and chaotic weighted backward shifts defined on general sequence spaces were obtained in [6,7], and characterizations for chaotic perturbations of the identity by weighted backward shifts are also in [7]. Other results for the linear dynamics of operators of the form $P(T)$, where T is an operator and $P(z)$ is a polynomial or a more general function, can be found in [8–13].

DeLaubenfels and Emamirad [10] proved that for a given non-constant polynomial $P(z)$, $P(B)$ is chaotic on ℓ^p, $1 \leq p < \infty$, whenever $P(\mathbb{D})$ intersects the unit circle [10, Th. 2.8], where $\mathbb{D}$ is the open unit disc of $\mathbb{C}$. Besides, for $a, b \in \mathbb{K}$, $\mathbb{K}$ being the scalar field, they showed that $aI + bB$ is chaotic if and only if $|b| > |1 - |a||$. Further sufficient conditions for chaos of $P(B)$ in terms of the coefficients of the polynomial had been stated in [9]. The aim of this note is to keep going on the search of 'computable' conditions for chaos of $\varphi(B)$, where $\varphi(z)$ is an analytic function. In this case, we deal with Linear Fractional Transformations, that is, analytic functions of the form $\varphi(z) = (az + b)/(cz + d)$. We prove that $\varphi(B)$ is chaotic on ℓ^p if and only if

$$\left| |d|^2 - |c|^2 - \left| b\bar{d} - a\bar{c} \right| \right| < |bc - ad| \,,$$

which obviously generalizes the above mentioned result in [10]. As a consequence, we also characterize chaotic weighted backward shifts on weighted ℓ^p spaces, result that serves us to know which operators of the form $\varphi(D)$ are chaotic on certain Banach spaces of analytic functions, where D is the differentiation operator. Although it will be shown later, we would like to point out that all operator considered in this paper are in fact (upper triangular) Toeplitz operators.

Let us start setting our frame of work. For a strictly positive weight sequence $v := (v_n)_n$, let

$$\ell^p(v) := \left\{ (x_n)_n \in \mathbb{C}^{\mathbb{N}}, \ \|x\|^p := \sum_{n=1}^{\infty} |x_n|^p \, v_n < \infty \right\}, \ 1 \leq p < \infty,$$

be the associated weighted ℓ^p-space. It is easy to check that condition

$$\sup_i \frac{v_i}{v_{i+1}} < \infty \tag{1}$$

characterizes boundedness of $B : \ell^p(v) \to \ell^p(v)$. This condition will always be assumed to hold. If the weighting sequence v coincides with the sequence of ones, the corresponding space will be denoted as ℓ^p.

Our aim is to state the chaotic behaviour of $\varphi(B)$ on ℓ^p, where φ is a Linear Fractional Transformation. In order to avoid trivial cases when $\varphi(z)$ reduces to a constant or to a degree 1 polynomial, we will assume that $ad - cb \neq 0$ and $c \neq 0$. Therefore, by saying that φ is a Linear Fractional Transformation (LFT for short) we mean

$$\varphi(z) = \frac{az + b}{cz + d}, \ a, b, c, d \in \mathbb{C}, \ ad - cb \neq 0, \ c \neq 0. \tag{2}$$

There are several ways to describe how the operator $\varphi(B)$ is defined but we will only speak here about two of them. The first one is to recall that the spectrum of B (i.e., the set of $\lambda \in \mathbb{C}$ such that $\lambda I + B$ is not invertible) is the closed unit disc $\overline{\mathbb{D}}$. Now, if we impose that $|d/c| > 1$ we have $\varphi(B) = (aB + bI)(cB + dI)^{-1}$ is a well defined bounded operator on ℓ^p. Our second approach defining $\varphi(B)$ is to use the Taylor expansion (around the origin) of φ, which turns out to be

$$\frac{b}{d} + \sum_{n=1}^{\infty} (-1)^{n+1} \, \frac{ad - bc}{cd} \left(\frac{c}{d} \right)^n z^n.$$

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If $|d/c| > 1$, then for each $x \in \ell^p$, we have

$$\sum_{n=1}^{\infty} \left(\frac{c}{d}\right)^n \|B^n x\| \leq \sum_{n=1}^{\infty} \left(\frac{c}{d}\right)^n \|x\| < \infty.$$

Therefore the series

$$\frac{b}{d} I + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{ad-bc}{cd} \left(\frac{c}{d}\right)^n B^n$$

converges pointwise on ℓ^p and we denote the limit operator as $\varphi(B)$, which is bounded by the Banach–Steinhauss Theorem (see, e.g., Appendix A in [3]).

At this point several observations should be made: 1) Functional Calculus Theory ensures that both approaches give the same operator. 2) The first approach is more direct but by the second one, it is easy to observe that if $\lambda \in \mathbb{C}$ is an eigenvalue of B, then $\varphi(\lambda)$ is an eigenvalue of $\varphi(B)$. 3) The second approach also shows that $\varphi(B)$ is a Toeplitz operator with canonical matrix

$$\begin{pmatrix} \frac{b}{d} & -\frac{ad-bc}{cd}\frac{c}{d} & \frac{ad-bc}{cd}\left(\frac{c}{d}\right)^2 & \cdots & \cdots & \cdots \\ 0 & \frac{b}{d} & -\frac{ad-bc}{cd}\frac{c}{d} & \frac{ad-bc}{cd}\left(\frac{c}{d}\right)^2 & \cdots & \cdots \\ 0 & 0 & \frac{b}{d} & -\frac{ad-bc}{cd}\frac{c}{d} & \frac{ad-bc}{cd}\left(\frac{c}{d}\right)^2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}.$$

To get chaos for $\varphi(B)$ on ℓ^p we will use the well known Eigenvalue Criterion for chaos (see any of the monographs [2,3] for a proof of it, and [14,15,10,12,9] for examples using it):

Theorem 1 *(Eigenvalue Criterion). Let $T : X \to X$ be an operator on a separable complex Banach space X. Suppose that the subspaces*

$$X_0 := \mathrm{Span}\{x \in X \ ; \ Tx = \lambda x \text{ for some } \lambda \in \mathbb{C} \text{ with } |\lambda| < 1\},$$
$$Y_0 := \mathrm{Span}\{x \in X \ ; \ Tx = \lambda x \text{ for some } \lambda \in \mathbb{C} \text{ with } |\lambda| > 1\},$$
$$Z_0 := \mathrm{Span}\{x \in X \ ; \ Tx = e^{\alpha\pi i} x \text{ for some } \alpha \in \mathbb{Q}\}$$

are dense in X, then T is chaotic.

In this framework, since the point spectrum (i.e., the set of eigenvalues) $\sigma_p(B) = \mathbb{D}$, this Criterion reads as $\varphi(B)$ is chaotic on ℓ^p if and only if $\varphi(\mathbb{D})$ intersects the unit circle. The following result gives a full geometrical description of $\varphi(\mathbb{D})$ when the pole of φ lies outside the closed unit disc. This result is already known since it might be used to characterize when a LFT maps the unit disk into itself [16,17]. For the sake of completeness, we provide here an elementary proof.

Lemma 2. *Let φ be a LFT as in (2) and $|d| > |c|$. Then $\varphi(\mathbb{D})$ is the disc $P + r\mathbb{D}$ with center P and radius r given by*

$$P = \frac{b\bar{d} - a\bar{c}}{|d|^2 - |c|^2}, \quad r = \frac{|bc - ad|}{|d|^2 - |c|^2}.$$

Proof. First we recall that LFT's map circles and lines to circles and lines. Since $\overline{\mathbb{D}}$ is, obviously, a bounded convex set, and the pole $-d/c$ lies outside the unit disc, we have that $\varphi(\overline{\mathbb{D}})$ must also be bounded and convex,

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therefore $\varphi(\overline{\mathbb{D}})$ is a circle whose boundary is $\varphi(\partial\mathbb{D})$. Now take three distinct points in the unit circle, for instance $z_1 = 1$, $z_2 = -1$, and $z_3 = i$. Since φ is a one to one transformation, we have that $A := f(z_1)$, $B := f(z_2)$, and $C := f(z_3)$ are three distinct points in the circle $\varphi(\partial\mathbb{D})$, that is, the circumscribed circle passing through A, B, and C in fact coincides with $\varphi(\partial\mathbb{D})$. We just need to show that

$$|A - P| = |B - P| = |C - P| = r.$$

Indeed, by using the well-known equalities $|z|^2 = z\bar{z}$ and $|c + d| = |\bar{c} + \bar{d}|$, we have

$$\begin{aligned}
|A - P| &= \left|\frac{a+b}{c+d} - \frac{b\bar{d} - a\bar{c}}{|d|^2 - |c|^2}\right| \\
&= \frac{1}{|d|^2 - |c|^2}\left|\frac{(a+b)(|d|^2 - |c|^2) - (b\bar{d} - a\bar{c})(c+d)}{c+d}\right| \\
&= \frac{1}{|d|^2 - |c|^2}\left|\frac{ad\bar{d} - bc\bar{c} - bc\bar{d} + ad\bar{c}}{c+d}\right| = r.
\end{aligned}$$

Analogously, noticing that $|ci + d| = |\bar{c} + \bar{d}i|$, we get

$$\begin{aligned}
|B - P| &= \left|\frac{-a+b}{-c+d} - \frac{b\bar{d} - a\bar{c}}{|d|^2 - |c|^2}\right| \\
&= \frac{1}{|d|^2 - |c|^2}\left|\frac{(-a+b)(|d|^2 - |c|^2) - (b\bar{d} - a\bar{c})(-c+d)}{-c+d}\right| \\
&= \frac{1}{|d|^2 - |c|^2}\left|\frac{-ad\bar{d} - bc\bar{c} + bc\bar{d} + ad\bar{c}}{-c+d}\right| = r. \\
|C - P| &= \left|\frac{ai+b}{ci+d} - \frac{b\bar{d} - a\bar{c}}{|d|^2 - |c|^2}\right| \\
&= \frac{1}{|d|^2 - |c|^2}\left|\frac{(ai+b)(|d|^2 - |c|^2) - (b\bar{d} - a\bar{c})(ci+d)}{ci+d}\right| \\
&= \frac{1}{|d|^2 - |c|^2}\left|\frac{ad\bar{d}i - bc\bar{c} - bc\bar{d}i + ad\bar{c}}{ci+d}\right| = r. \qquad \square
\end{aligned}$$

Theorem 3. *Let φ be a LFT as in (2) and $|d| > |c|$. The operator $\varphi(B)$ is chaotic on ℓ^p if and only if*

$$\left||d|^2 - |c|^2 - \left|b\bar{d} - a\bar{c}\right|\right| < |bc - ad|.$$

Proof. As in the previous lemma, denote $\varphi(\mathbb{D}) = P + r\mathbb{D}$. We have that $\varphi(B)$ is chaotic if and only if $P + r\mathbb{D}$ intersects the unit circle. In order to accomplish that, we have two possibilities: if the center P lies inside the unit disc, then $|P| + |r| > 1$; on the other hand, if P lies outside the closed unit disc, then $|P| - |r| < 1$. Both conditions lead us to

$$-|r| < 1 - |P| < |r|,$$

and the conclusion follows after substituting the values for P and r and working out the algebra. $\square$

Corollary 4. *Let φ be a LFT as in (2), $|d| > |c|$, and $g(z) = z^n$, with n any positive integer. Consider the composition $\varphi \circ g(z) = (az^n + b)/(cz^n + d)$, then the operator $\varphi \circ g(B)$ is chaotic on ℓ^p if and only if*

$$\left||d|^2 - |c|^2 - \left|b\bar{d} - a\bar{c}\right|\right| < |bc - ad|.$$

Proof. We just need to observe that $\varphi(\mathbb{D})$ intersects the unit circle if, and only if, so does $\varphi \circ g(\mathbb{D})$. □

Next, we want to study chaos for $\varphi(B)$ defined on weighted $\ell^p(v)$ spaces. It is easy to see that $\varrho\mathbb{D} \subset \sigma_p(B)$, where $\varrho^p = \liminf_i {v_i}^{-1/i}$. Unfortunately, in this framework, chaos of $\varphi(B)$ is not equivalent to $\varphi(\varrho\mathbb{D}) \cap \partial\mathbb{D} \neq \emptyset$, not even for the simple case $f(z) = z$: if we take the spaces ℓ^1, $\ell^1(1/n)$, and $\ell^1(1/n^2)$ (by certain abuse of notation, $\ell^1(1/n) = \ell^1(v)$ and $\ell^1(1/n^2) = \ell^1(v')$, where $v := (1/n)_n$ and $v' := (1/n^2)_n$, respectively), in all three cases $\varrho = 1$ and $f(\mathbb{D}) \cap \partial\mathbb{D} = \emptyset$. However B is not hypercyclic on ℓ^1, it is hypercyclic but not chaotic on $\ell^1(1/n)$, and it is chaotic on $\ell^1(1/n^2)$ (see, e.g., Chapter 4 in [3]). Aware of the previous situation, the Eigenvalue Criterion 1 ensures us that if $\varphi(\varrho\mathbb{D}) \cap \partial\mathbb{D} \neq \emptyset$, then $\varphi(B)$ is chaotic on $\ell^p(v)$.

Before we state the theorem providing chaos for $\varphi(B)$ on $\ell^p(v)$, let us remark that $\varrho < \infty$. Indeed, the weight condition (1) implies that there exists $M > 0$ such that for each $i \geq 2$ we have $v_i > v_{i-1}/M$. Inductively $v_i > v_1 M^{1-i}$ for all $i > 1$ and therefore $\varrho^p \leq M < \infty$. Also, we disregard $\varrho = 0$. To see why, first write $\varphi(B) = \varphi(0)I + \phi(B)$, where ϕ is an holomorphic function with $\phi(0) = 0$. By the Eigenvalue Criterion 1, the operator $T = \varphi(B)$ has a chance to be hypercyclic only if $|\varphi(0)| = |b/d| = 1$, but since $\varrho = 0$, we end up with a compact perturbation of $(b/d)I$ (i.e., $T = (b/d)I + K$, where $K(B_X)$ is relatively compact), which are never chaotic (see [7, Proposition 6.1]). Indeed, if T admits a dense set of periodic points, then the unique eigenvalue b/d of T is certain n-root of 1, thus $T^n = I + K'$ for certain compact operator K', and T^n admits a dense set of periodic points too, which contradicts [7, Proposition 6.1].

Theorem 5. *Let $v = (v_i)_i$ be a sequence satisfying the weight condition (1). Set $\varrho^p = \liminf_i {v_i}^{-1/i}$ and let φ be a LFT as in (2) with $|d| > \varrho\,|c|$. If*

$$\left| |d|^2 - \varrho^2\,|c|^2 - \left|b\bar{d} - \varrho^2 a\bar{c}\right| \right| < \varrho\,|bc - ad|\,,$$

then the operator $\varphi(B)$ is chaotic on $\ell^p(v)$.

Proof. Since $\varrho\mathbb{D} \subset \sigma_p(B)$, we have to check that $\varphi(\varrho\mathbb{D})$ meets the unit circle. Take the homothety $g(z) = \varrho z$ and observe that $\varphi(\varrho\mathbb{D}) = \varphi \circ g(\mathbb{D})$. Now, apply Lemma 2 to the composition $\varphi \circ g(z) = (\varrho az + b)/(\varrho cz + d)$ and follow a similar argument to Theorem 3. □

Remark 6. Some operators can be represented as a weighted backward shift operator $B_w(x_1, x_2, \dots) := (w_2x_2, w_3x_3, \dots)$ defined on a weighted $\ell^p(v)$ space. This case may be reduced to the previous one via topological conjugacy. Set

$$a_1 := 1,\ a_i := w_2 \dots w_i,\ i > 1,$$

and consider $\ell^p(\bar{v})$ where

$$\bar{v}_i = \frac{v_i}{\prod_{j=2}^{i} w_j^p},\ \text{for all } i.$$

Take $\phi_a : \ell^p(v) \to \ell^p(\bar{v})$ defined as $\phi_a(x_1, x_2, \dots) := (a_1x_1, a_2x_2, \dots)$ to construct a commutative diagram $\phi_a \circ B_w = B \circ \phi_a$. Since ϕ_a is an isometry, $\|x\|_{\ell^p(v)} = \|\phi_a(x)\|_{\ell^p(\bar{v})}$, by topological conjugation we have that $\varphi(B)$ is chaotic on $\ell^p(\bar{v})$ if and only if $\varphi(B_w)$ is chaotic on $\ell^p(v)$ (notice that $B^n = (\phi_a \circ B_w \circ \phi_a^{-1})^n = \phi_a \circ B_w^n \circ \phi_a^{-1}$ and use the Taylor expansion of φ).

LFT's of differential operators on Hilbert spaces of entire functions (see [18]). Let $\gamma(z)$ be an admissible comparison entire function, that is, the Taylor coefficients $\gamma_j > 0$ for all $j \in \mathbb{N}_0$ and the sequence $(j\gamma_j/\gamma_{j-1})_{j\geq 1}$ is monotonically decreasing. We consider the Hilbert space $E^2(\gamma)$ of power series

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$$g(z) = \sum_{j=0}^{\infty} \hat{g}(j) z^j$$

for which

$$\|g\|_{2,\gamma}^2 := \sum_{j=0}^{\infty} \gamma_j^{-2} |\hat{g}(j)|^2 < \infty.$$

Chan and Shapiro showed that for $a \neq 0$ the translation operator T_a is hypercyclic on $E^2(\gamma)$ (see [18, Theorem 2.1]). They proved that $T_a = \sum_{n\geq 0} a^n/n!\ D^n$, where D is the operator of differentiation. They pointed out that the hypercyclicity of T_a is in fact the hypercyclicity of $\varphi(D)$ for the particular case $\varphi(z) = e^{az}$. They also asked for the dynamics of 'other' operators; we consider here operators of the form $\varphi(D)$, where $\varphi(z)$ is a LFT.

It is clear that $E^2(\gamma)$ is isometric to $\ell^2(v)$ with $v = (v_j)_{j\in\mathbb{N}_0} = (\gamma_j^{-2})_{j\in\mathbb{N}_0}$ and with the identification $f \mapsto (f^{(j)}(0)/j!)_{j\geq 0}$. Moreover, the operator of differentiation D turns out to be a weighted backward shift with weights $w = (w_j)_{j\geq 1} = (j)_{j\geq 1}$ or, equivalently, as a backward shift defined on $\ell^2(\bar{v})$, where

$$\bar{v}_j = \frac{1}{(\gamma_j\ j!)^2}, \quad j \geq 0.$$

Since $\gamma(z)$ is an admissible comparison entire function, it is easy to check that the weight condition (1) is satisfied and $\varphi(B)$ is a bounded operator on $\ell^2(\bar{v})$ for any LFT φ with $|d| > \varrho\,|c|$ (this is equivalent to saying that $\varphi(D)$ is a bounded operator on $E^2(\gamma)$ for any LFT φ with $|d| > \varrho\,|c|$).

Let us now consider only those spaces for which $\lim_{j\to\infty} j\gamma_j/\gamma_{j-1} > 0$. Since the limit of successive roots coincides with the limit of successive ratios we have that $\varrho = \lim_{j\to\infty} j\gamma_j/\gamma_{j-1} > 0$, and Theorem 5 may be applied to LFT's φ with $|d| > \varrho\,|c|$.

To focus on a concrete example take $\gamma(z) = e^{\alpha z}$ with $\alpha > 0$; it is clear that $\varrho = \alpha$. If

$$\left| |d|^2 - \alpha^2 |c|^2 - \left| b\bar{d} - \alpha^2 a\bar{c} \right| \right| < \alpha\, |bc - ad|\,,$$

then $\varphi(D)$ is chaotic on $E^2(e^{\alpha z})$, where φ is a LFT with $|d| > \varrho\,|c|$.

Backward shift operators on spaces of analytic functions. Consider spaces consisting on formal power series $\phi(z) := \sum_{j=0}^{\infty} \hat{f}_j z^j$ for which the sequence $(\hat{f}_j)_j$ belongs to $\ell^2(v)$ for a certain weight sequence $v = (v_j)_j$. For example, the Hardy space H^2 is obtained for $v = (1)_j$, the Bergman space A^2 for $v = (1/(1+j))_j$, and the Dirichlet space $\mathcal{D}$ for $v = (j+1)_j$. For the orthogonal basis $(z^j)_{j\geq 0}$, the backward shift is defined as $B(z^j) := z^{j-1}$ for $j \in \mathbb{N}$, and $B(1) = 0$. Now, if φ is a LFT with $|d| > |c|$, since $\varrho^2 = \liminf_j v_j^{-1/j} = 1$ for the spaces H^2, A^2, and $\mathcal{D}$, chaos of $\varphi(B)$ may be obtained by using Theorem 3.

Remark 7. In this article we only considered the notion of chaos in the sense of Devaney, but other forms of chaos, like the existence of mixing invariant probability measures with full support (see [19–21]), or distributional chaos (see [22]) also happen under the same conditions presented here for Devaney chaos.

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